Bank Capital, Government Bond Holdings, and Sovereign Debt Capacity*

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Abstract

I develop a model where the sovereign debt capacity depends on the capitalization of domestic banks. Low-capital banks optimally tilt their government bond portfolio toward domestic securities, linking their destiny to that of the sovereign. If the sovereign risk is sufficiently high, low-capital banks lend less to the productive sector to further increase their holdings of domestic government bonds, lowering sovereign yields. In this case, a government that regulates bank capital faces a trade-off. On the one hand, high-capital banks lend more to the productive sector. On the other hand, low-capital banks support the home sovereign debt capacity.

JEL Codes: E44, F33, G21, G28.

Keywords: Bank Capital, Sovereign Risk, Government Bonds, Bank-Sovereign Nexus.

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1 Introduction

Banks hold a large share of their assets in domestic government bonds and rely on government guarantees. As a result, the credit risks of sovereigns and banks are dangerously intertwined. During crises, the increased sovereign credit risk impairs the balance sheets of banks that, in turn, rely on the domestic government for a bailout. While the literature has thoroughly analyzed how this “diabolic loop” materializes during bad times, little work has been done to understand what determines, *ex-ante*, the link between sovereign and financial credit risks. My goal is to provide a theory to fill this gap.

In this paper, I build a tractable model where the sovereign debt capacity depends on the capitalization of domestic banks. Protected by deposit insurance, low-capital banks hold more domestic than foreign government bonds to link their destiny to that of the home sovereign. Banks risk-shift using domestic government bonds, as these assets promise the highest payoff in the good state, and limited liability protects banks’ equity holders in case of sovereign default. If the sovereign risk is sufficiently high, risk-shifting banks lend less to the productive sector to hold even more domestic government bonds, lowering sovereign yields and effectively supporting public debt issuance.

The model features two countries and two dates. Each country has a productive sector, a financial sector, and a government. The productive sector borrows from the financial sector and invests in a productive technology. The financial sector invests in loans to the productive sector, in domestic government bonds, and in foreign government bonds. The government decides the initial level of bank debt, provides a public good funded by one-period bonds, and repays bondholders at \( t = 1 \) applying an exogenous tax rate to the payoff of the productive technology. This payoff is stochastic. In the good state, the high payoff allows the productive sector to fully repay the loan to banks and the government collects enough taxes to fully repay bondholders. In the bad state, the low payoff forces the productive sector and the government to default. The payoff of the government is made of three terms: (i) the benefit
from the provision of the public good funded by the sovereign debt, (ii) the payoff generated by the private (productive and financial) sector, and (iii) the cost of bank default. Bank depositors are protected by a credible deposit insurance.

Banks’ government bond portfolio choice depends on whether the limited liability constraint binds in the bad state. If it does not bind, banks are “well capitalized” and invest in both domestic and foreign government bonds. If it binds, banks are “undercapitalized” and develop a preference, within the government bond portfolio, for domestic bonds. These assets perform well in the good state and poorly in the bad state, exactly when their payoff is entirely used to pay depositors. As shocks are not perfectly correlated across countries, this is not the case for foreign bonds as their payoff depends on the state of the foreign country.

If the sovereign risk is sufficiently high, domestic government bonds become an even more attractive asset to risk-shift. In this environment, undercapitalized banks invest less in lending to the productive sector to hold more domestic government bonds. In equilibrium, in a country with undercapitalized domestic banks, (i) the sovereign has a higher debt capacity and pays lower sovereign rates and (ii) the productive sector invests less in its productive technology and pays higher corporate loan rates compared with the case where the country’s banks are well capitalized.

Governments face a trade-off when setting capital regulation in a country with sufficiently high sovereign risk. On the one hand, well capitalized domestic banks choose a high loan to the productive sector, thereby supporting a high investment in the productive technology. On the other hand, undercapitalized domestic banks choose a high investment in domestic government bonds, expanding the sovereign debt capacity that, in turn, supports the public good provision. If the benefit from higher public spending offsets the lower payoff generated by the productive sector funded by undercapitalized banks, the government chooses a level of bank debt to minimize the cost of bank default, just enough to induce domestic banks to risk-shift using domestic bonds.

The model relies on three key assumptions. The first is a sufficiently high correlation be-
tween the payoff of domestic government bonds and the payoff of the productive technology to ensure that weak banks default when the domestic sovereign defaults. The second assumption is a bank balance sheet with a fixed size—an extreme version of funding constraint—that links the holdings of domestic government bonds to the investment in the productive technology. The third is the presence of a credible deposit insurance (or any other guarantee) protecting bank depositors that, in turn, do not require a high return on their savings nor discipline banks by withdrawing their deposits.

The insights of the model can be applied to the eurozone because of (i) the high covariance between bank and sovereign risks mainly caused by the absence of a eurozone-wide safe asset and (ii) the role of the European Central Bank (ECB) as a supranational safety net for banks. In this context, I provide empirical evidence consistent with the proposed mechanism. Using eurozone publicly available stress test data in December 2010—before the worsening of the eurozone crisis—I show that (i) several banks in “peripheral” countries (Greece, Ireland, Italy, Portugal, Spain) had “solvency-critical” exposures toward their domestic sovereign and (ii) peripheral banks held more domestic government bonds, driven by low-capital banks, compared with non-peripheral banks.

I contribute to the literature by presenting a new theory, based on the well-known risk-shifting motive (Jensen and Meckling, 1976), linking ex-ante holdings of domestic government bonds and sovereign risk. By adding a key role for bank capital, I contribute to the literature on the repatriation of sovereign debt in bad times (Broner et al., 2014, 2010; Gennaioli et al., 2018), typically based on the role of secondary markets or selective sovereign defaults. I also complement the literature on the banks-sovereign nexus (Acharya et al., 2014; Brunnermeier et al., 2016; Cooper and Nikolov, 2018; Farhi and Tirole, 2018; Leonello, 2018) by shifting the focus from the link running from banks’ balance sheets to the government to ex-ante
My findings also relate to the literature on bank private credit supply following a large increase in sovereign risk. The empirical literature on the eurozone crisis documents a negative correlation between bank holdings of risky sovereign bonds and subsequent change in bank credit supply (Acharya et al., 2018; Bofondi et al., 2018; Bottero et al., 2020; De Marco, 2019; Popov and Van Horen, 2015). Following Bocola (2016) and Perez (2018), this finding is interpreted as the effect of a shock to the value of the government bond portfolio on bank funding costs. My analysis highlights that, because of risk-shifting, these losses might be concentrated on the balance sheet of the weakest banks (that ex-ante hold more domestic government bonds), making sovereigns and banks more vulnerable during bad times.

Finally, my analysis also relates to the literature on cross-border regulation (Beck and Wagner, 2016; Dell’Ariccia and Marquez, 2006; Loranth and Morrison, 2007) and the literature on the effect of capital (Caballero et al., 2008; Diamond and Rajan, 2011) and government guarantees (Allen et al., 2018) on bank portfolio choice.

The remainder of the paper is organized as follows. In Section 2, I illustrate the model setup and define the equilibrium concept. In Section 3, I present the model solution. In Section 4, I discuss the model assumptions, applications, and extensions. In Section 5, I present supportive empirical evidence. Concluding remarks are given in Section 6.

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1 The large literature on holdings of domestic government bonds during crises attributes it to a flight to safety (Caballero and Farhi, 2013), the collateral role for interbank loans (Bolton and Jeanne, 2011), the collateral eligibility at the central bank (Uhlig, 2013), the lack of bank balance sheet transparency (Ari, 2017), government guarantees (Koetter and Popov, forthcoming), and government moral suasion (Becker and Ivashina, 2018; Ongena et al., 2019).

2 Almeida et al. (2017) document a new “credit ratings” channel by showing that sovereign risk has a negative effect on the real economy following a sovereign rating downgrade.
2 Model

In this section, I set up the model and define the equilibrium concept.

2.1 Setup

There are two dates: $t = 0$ and $t = 1$. There are two symmetric countries $i \in \mathcal{I}$, where $\mathcal{I} = \{A, B\}$. Each country has a productive sector, a financial sector, and a government. There is universal risk neutrality and no discounting. There are two states of the world $s^i \in S$ at $t = 1$ in each country, where $S = \{H, L\}$. I describe the setup for one country, omitting for simplicity the country superscripts. Figure 1 illustrates the timeline of the economy.

**Productive Sector** There is one representative entrepreneur that borrows from the financial sector to invest in a productive technology. The productive technology can be hit by a negative shock between $t = 0$ and $t = 1$. An investment of $k$ at $t = 0$ yields $\epsilon_H \sqrt{k}$ with probability $\theta$ and $\epsilon_L \sqrt{k}$ with probability $1 - \theta$ at $t = 1$, where $\theta \in (0, 1)$. I assume $\theta \epsilon_H > (1 + \theta) \epsilon_L$ to ensure that the productive sector defaults on the loan in the bad state—the economically interesting case for the analysis. The payoff of the productive technology is taxed at a rate $\tau$. The problem of “entrepreneurs” at $t = 0$ is:

$$
\max_k \mathbb{E}(\epsilon)(1 - \tau)\sqrt{k} - \mathbb{E}(\lambda^C)R^C k
$$

(1)

where $R^C$ is the corporate loan rate and $\lambda^C$ is the recovery rate on the corporate loan.

**Financial Sector** There is one representative bank with a balance sheet of size one, debt $L \in (0, 1)$ maturing at $t = 1$, and equity $1 - L$. It maximizes profits by investing in a corporate loan to the productive sector, in domestic government bonds, and in foreign government bonds. The financial sector invests $\tilde{k}$ in the corporate loan, $\alpha(1 - \tilde{k})$ in domestic government bonds, and $(1 - \alpha)(1 - \tilde{k})$ in foreign government bonds. The variable $\alpha \in [0, 1]$
Figure 1: Timeline. This figure illustrates the timeline of the economy for one country. It captures the “home bias” of the financial sector in the bond portfolio. If $\alpha = 1$, “banks” invest only in domestic bonds. If $\alpha = 0$, banks invest only in foreign bonds. The financial sector is protected by limited liability. Banks’ problem at $t = 0$ is:

$$\max_{\alpha, k} E \left( \left[ \Pi - L \right]^+ \right)$$

where

$$\Pi_{s, s^*} = \tilde{k}\lambda^C R^C + \alpha(1 - \tilde{k})\lambda_s R + (1 - \alpha)(1 - \tilde{k})\lambda_{s^*} R^*$$

where $s \in S$ is a state of the world, $R$ is the gross interest rate paid by government bonds, $R^C$ is the gross interest rate paid by the corporate loan, $\lambda$ is the recovery rate of government bonds, and $\lambda^C$ is the recovery rate of the corporate loan. The star indicates a foreign variable.

Bank depositors hold bank debt $L$ and are protected by a supranational deposit insurance. Depositors and the deposit insurance are unmodeled. As opposed to a nationally funded deposit insurance, the supranational deposit insurance is able to credibly protect
depositors in case of domestic sovereign default.\footnote{One example of a supranational deposit insurance is the European Deposit Insurance Scheme (EDIS). In Section 4, I discuss the role that the deposit insurance plays in the model in greater detail.} Because depositors do not suffer losses caused by an eventual bank default, the return on bank deposits is one.

**Government** The government plays two roles. First, it provides a public good funded by one-period bonds issued at $t = 0$. The government collects taxes and repays bondholders at $t = 1$, applying an exogenous tax rate $\tau$ to the payoff of the productive technology. Second, the government acts as a regulator of bank capital deciding—before banks make their investment decision at $t = 0$—the level of initial bank debt $L$. The problem of the government at $t = 0$ is:

$$\max_{L,D} \mathbb{E} \left( g(D) + \Pi^P(L) - h(L)\mathbf{I}_{def} \right) \quad s.t. \quad D \leq \overline{D}(L) \quad (3)$$

The objective function is made of three terms. The first term is the benefit from the provision of the public good funded by the sovereign debt $D$, where $g$ is a strictly increasing function. The second term is the payoff generated by the private (financial and productive) sector.\footnote{The payoff $\Pi^P$ is split between entrepreneurs, bank depositors, and bank equity holders. If the financial sector defaults, bank equity holders get zero and bank depositors get the full bank payoff plus the payment from the supranational deposit insurance.} The third term is the cost of bank failure, strictly positive in case of default ($\mathbf{I}_{def} = 1$) and increasing in bank debt with $h(0) = 0$. Given that the objective function is increasing in the level of sovereign debt $D$, the government exhausts its debt capacity setting $D = \overline{D}(L)$.

### 2.2 Sovereign Debt Capacity

I now illustrate how the sovereign might default and derive its debt capacity.
**Sovereign Default** The sovereign defaults when tax collection is too low to fully repay bondholders. There is no strategic default, as the government, conditional on having sufficient funds, always repays its debt. The tax collection is subject to a (sovereign) shock: part of the tax collection disappears and therefore cannot be used to repay bondholders at $t = 1$, causing, in equilibrium, an increase in the sovereign borrowing cost. This shock can be seen as a series of populist measures that politicians might adopt to be re-elected, thereby wasting part of the tax collection that could instead be used to repay bondholders. Conversely, during the eurozone crisis, some countries appointed “technocratic” governments led by non-politicians in an attempt to reduce their borrowing costs. One example is the appointment of Monti’s government in Italy in November 2011 to succeed Berlusconi’s coalition government—a development immediately followed by a drop in Italian sovereign yields.

In case of default, the government applies an haircut $1 - \lambda$ to its payments to bondholders. If the recovery rate $\lambda < 1$, the government defaults, being able to repay only a fraction $\lambda$ of the payments due. If the recovery rate $\lambda = 0$, the government defaults on the entire debt. The payoff from the productive technology is split between the sovereign (tax collection) and entrepreneurs (after-tax revenues) that, in turn, use it to repay the bank loan. The uncertain tax collection, after being hit by the shock $y$, is then used by the government to repay bondholders.

**Sovereign Debt Capacity** Banks anticipate that the government might default and constrain its public debt issuance accordingly. In particular, I assume, as in Acharya and Rajan (2013), that banks are willing to invest in public debt if the payments due to bondholders are less than or equal to the expected tax collection minus the sovereign shock $y$.\(^5\)

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\(^5\)I focus on the economically interesting case where the sovereign shock erodes only part of the tax collection at $t = 1$, namely $y < \tau \epsilon L \sqrt{k}$. In the appendix, I derive this condition in terms of primitives.
\[ DR \leq \mathbb{E}(\epsilon) \tau \sqrt{k} - y \]

Rearranging, I obtain the sovereign debt capacity:

\[ \overline{D} = \frac{\mathbb{E}(\epsilon) \tau \sqrt{k} - y}{R} \]

This expression for the sovereign debt capacity is consistent with the idea that sovereigns might default if the cost of debt becomes “too high” because, for example, the benefit from not repaying their debt outweighs the cost of exclusion from international credit markets (Eaton and Gersovitz, 1981; Mendoza and Yue, 2012) or because future fiscal surpluses are insufficient to sustain the current sovereign debt (Bohn, 1998; Ghosh et al., 2013).

### 2.3 Equilibrium

Hereafter, I use the following definition of equilibrium:

**Definition 1.** Given \( g^i, h^i, \) tax rates \( \tau^i, \) probabilities \( \theta^i, \) productivity parameters \( \epsilon^i, \) and sovereign shocks \( y^i, \) where \( i \in I, \) an equilibrium is:

- gross returns on government bonds \( R^i \)
- gross returns on corporate loans \( R^{Ci} \)
- public debt issuance \( D^i \)
- recovery values on government bonds \( \lambda^i_s, \) for \( s^i \in S \)
- recovery values on corporate loans \( \lambda^{Ci}_s, \) for \( s^i \in S \)
- financial sectors’ investment decisions \( \alpha^i, \tilde{k}^i \)
- entrepreneurs’ investment decisions \( k^i \)
- levels of bank debt \( L^i \)

such that

- entrepreneurs solve (1)
According to corporate loan market clearing, the entrepreneurs’ investment in the productive technology is equal to the financial sectors’ investment in corporate credit:

\[ k^i = \bar{k}^i \quad i \in \mathcal{I} \]

The bond market clearing conditions for bonds issued by A and bonds issued by B are:

\[ \alpha^A(1 - k^A) + (1 - \alpha^B)(1 - k^B) = D^A \]
\[ \alpha^B(1 - k^B) + (1 - \alpha^A)(1 - k^A) = D^B \]

where the first term is the domestic demand for sovereign bonds and the second term is the foreign demand for sovereign bonds.

### 3 Bank Capital and Portfolio Choice

In this section, I solve the model using a backward induction argument. In Section 3.1 and Section 3.2, I derive the recovery values on corporate loans and government bonds taking the choices of entrepreneurs, the choices of banks, and prices as given. In Section 3.3, I solve banks’ problem taking their initial debt level \( L \) as given and obtain—using market clearing conditions—candidate equilibrium quantities and prices. In Section 3.4, I solve for the endogenous debt level that triggers the limited liability constraint to bind based on the candidate equilibrium quantities and prices above. In Section 3.5, I solve the government problem at \( t = 0 \) and obtain equilibrium quantities and prices.
3.1 Corporate Loan Default

At $t = 0$, entrepreneurs borrow from the financial sector to invest in the productive technology. Their loan demand is met by the financial sector, namely $k^i = \bar{k}^i$. Entrepreneurs use the payoff of the productive technology at $t = 1$ to pay taxes and repay the bank loan. As anticipated, the assumption $\theta \epsilon_H > \epsilon_L(1 + \theta)$ ensures that the entrepreneurs (i) repay the bank in full if the payoff from the productive technology is high and (ii) default if the payoff from the productive technology is low. Formally, omitting the country superscripts:

**Lemma 1.** Entrepreneurs only default in the bad domestic state, i.e. $\lambda^C_H = 1$ and $\lambda^C_L < 1$.

The corporate loan recovery rate in the bad state is:

$$\lambda^C_L = \frac{2\theta \epsilon_L}{\theta \epsilon_H - \epsilon_L(1 - \theta)} \in (0, 1)$$

The recovery rate in the bad state is a function of the probability $\theta$ and the productivity parameters $\epsilon_H$ and $\epsilon_L$. The corporate loan recovery rate in the bad state is (i) increasing in $\epsilon_L$ as a higher payoff in the bad state allows entrepreneurs to increase their repayment in that state and (ii) decreasing in the probability $\theta$ and in $\epsilon_H$ as a higher expected payoff and a higher payoff in the good state induce entrepreneurs to borrow more.

3.2 Sovereign Default

Governments exhaust their debt capacity choosing $D = \overline{D}$. While the tax collection is sufficient to repay bondholders in the good state, the government is forced to write-down part of its debt in the bad state. Formally, omitting the country superscripts:

**Lemma 2.** Governments only default in the bad domestic state, i.e. $\lambda_H = 1$ and $\lambda_L < 1$.

The recovery rate on sovereign debt in the bad state $\lambda_L$ depends on the sovereign shock $y$ and the tax collection that, in turn, depends on entrepreneurs’ investment $k$ in their
productive technology (the tax base). To gain tractability, I rewrite the sovereign shock $y$ as a fraction $\gamma \in (0, 1)$ of tax collection in the bad state of the world, namely:

$$y = \gamma \tau \epsilon L \sqrt{k}$$

I refer to $\gamma$ as the “sovereign risk” in the economy. Formally, the sovereign risk $\gamma$ depends on the size of the shock $y$ as well as the equilibrium investment $k$. The sovereign debt recovery rate in the bad state and the sovereign debt capacity can be written as:

$$\lambda_L = \frac{\epsilon_L (1 - \gamma)}{\Delta \epsilon} \in (0, 1)$$

and

$$\overline{D} = \frac{\Delta \epsilon \tau \sqrt{k}}{R}$$

where $\Delta \epsilon = \theta \epsilon_H + (1 - \theta - \gamma) \epsilon_L$.

### 3.3 Bank Portfolio Choice

In this subsection, I solve banks’ problem for a given initial debt level $L$ and obtain candidate equilibrium quantities and prices using market clearing conditions. In Section 3.3.1, I assume that the limited liability constraint does not bind for either of the two financial sectors (both are “well capitalized”). In Section 3.3.2, I assume that the limited liability constraint binds for one or both financial sectors (one or both are “undercapitalized”). In Section 3.3.3, I compare these candidate equilibria.

To isolate the role of bank capital, I assume that the two countries are identical except for their initial level of bank debt $L$. Depending on the two financial sectors’ levels of debt, there are four cases: WW, UW, WU, or UU. The first (second) letter refers to whether banks in country A (B) are well capitalized or undercapitalized. For example, UW corresponds to

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6In the appendix, I formally show how that $\gamma$ is an increasing function of $y$. 

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the case where country A’s financial sector is undercapitalized and country B’s financial sector is well capitalized.

3.3.1 Well Capitalized Banks

The two financial sectors, if both are well capitalized, invest in the two government bond markets and have the same home bias in equilibrium.

**Proposition 1.** In the WW case, $\alpha^A = \alpha^B$.

In the WW case, the two financial sectors solve the same problem. By symmetry, they invest $k^A = k^B = k_W$ in the corporate loan and allocate the same share $\alpha^A = \alpha^B = \alpha_W$ of the remaining unit balance sheet capacity to domestic government bonds. Entrepreneurs invest $k^A = k^B = k_W$ in their productive technology and pay $R^{CA} = R^{CB} = R^C_W$ to borrow from the financial sectors. The gross return on government bonds is $R^A = R^B = R_W$ and public debt issuance is $D^A = D^B = D_W$.

The economy presents a continuum of equilibria characterized by different levels of banks’ home bias $\alpha_W \in [0, 1]$. In a high home bias equilibrium, financial sectors allocate the largest share of their government bond portfolio domestically. This is the type of equilibrium typically observed in the data. In a low home bias equilibrium, financial sectors invest the largest share of their government bond portfolio abroad. Quantities and prices do not depend on the home bias, which is indeterminate in equilibrium.

As the sovereign risk $\gamma$ increases, sovereign bonds become riskier, banks lend more to the productive sector, and the sovereign debt capacity goes down. If the sovereign risk is higher than a threshold level $\bar{\gamma}$, sovereign bonds become riskier than corporate bonds, namely the recovery rate of sovereign bonds is lower than the recovery rate on corporate bonds:

$$\gamma > \bar{\gamma} \iff \mathbb{E}(\lambda) < \mathbb{E}(\lambda^C)$$
I refer to this case as the “high sovereign risk” case.\textsuperscript{7}

The expected return from investing in government bonds is equal to the expected return from investing in the corporate loan, namely $\mathbb{E}(\lambda)R_W = \mathbb{E}(\lambda^C)R^C_W$. If the sovereign risk is low ($\gamma < \bar{\gamma}$), the return in the good state on government bonds $R_W$ is lower than the return in the good state on the corporate loan $R^C_W$. If the sovereign risk is high ($\gamma > \bar{\gamma}$), the return in the good state on government bonds $R_W$ is higher than the return in the good state on the corporate loan $R^C_W$ as the financial sector requires a higher compensation to invest in the risky public debt.

### 3.3.2 Undercapitalized Banks

If one or both financial sectors are undercapitalized (UW, WU, or UU case), banks develop a preference for domestic over foreign government bonds. In Figure 2, I illustrate how the limited liability constraint generates this home bias in the government bond portfolio. The figure shows the payoffs of bank equity holders at $t = 1$. The left and right panels show the payoffs of well capitalized and undercapitalized banks, respectively. If well capitalized, equity holders obtain the full payoff in the good state and the post-haircut payoff in the bad state. If undercapitalized, equity holders obtain the full payoff in the good state and zero in the bad state, exactly where their entire payoffs are used to repay depositors. In sum, the payoff of domestic government bonds only depends on the domestic state. As shocks are uncorrelated across countries, this is not the case for the payoff of foreign government bonds.

More formally, I obtain the decision about the allocation of government bonds taking partial derivatives in (2) with respect to $\alpha$:

\textsuperscript{7}Formally, $\bar{\gamma} = 1 - \frac{2\theta^2(\epsilon_H - \epsilon_L)}{\theta(\epsilon_H - \epsilon_L)(1+\theta)} < 1$. See the appendix for a formal derivation.
Well Capitalized Banks
Limited liability does not bind.

Undercapitalized Banks
Limited liability binds in the bad state.

Figure 2: Banks’ Payoffs and Capitalization. This figure shows the payoffs of well capitalized banks (left panel) and undercapitalized banks (right panel) at $t = 1$.

If well capitalized, banks choose

$$
\alpha = \begin{cases} 
1 & \text{if } \mathbb{E}(\lambda)R > \mathbb{E}^*(\lambda^*)R^* \\
0 & \text{if } \mathbb{E}(\lambda)R < \mathbb{E}^*(\lambda^*)R^* \\
\in [0,1] & \text{if } \mathbb{E}(\lambda)R = \mathbb{E}^*(\lambda^*)R^*
\end{cases}
$$

(5a)

If undercapitalized, banks choose

$$
\alpha = \begin{cases} 
1 & \text{if } R > \mathbb{E}^*(\lambda^*)R^* \\
0 & \text{if } R < \mathbb{E}^*(\lambda^*)R^* \\
\in [0,1] & \text{if } R = \mathbb{E}^*(\lambda^*)R^*
\end{cases}
$$

(5b)

where, using Lemma 2, $\mathbb{E}(\lambda) \in (0,1)$ and $\mathbb{E}^*(\lambda^*) \in (0,1)$. On the one hand, well capitalized banks invest in the government bonds with the highest expected return $\mathbb{E}(\lambda)R$. On the other hand, undercapitalized banks develop a preference for domestic government bonds and therefore need to be compensated to hold foreign government bonds.

**Proposition 2.** If one or both financial sectors are undercapitalized, $\alpha^A = \alpha^B = 1$.

In (the candidate) equilibrium, an undercapitalized financial sector invests its entire government bond portfolio domestically. Intuitively, undercapitalized banks want to risk-
shift on their insured depositors and therefore prefer assets yielding the highest payoff in the good domestic state and the lowest payoff in the bad domestic state. Within the government bond portfolio, domestic government bonds satisfy these requirements. As I illustrate in the next subsection, the choice between domestic government bonds and the corporate loan depends on the level of sovereign risk.

The high demand for domestic bonds by undercapitalized banks causes the foreign financial sector—regardless of its capitalization—to also invest its entire government bond portfolio domestically. Suppose, for example, that country A has undercapitalized banks and country B has well capitalized banks (UW case). Undercapitalized banks in country A invest more in domestic bonds, lowering their yield. Well capitalized banks in country B then tilt their government bond portfolio domestically, as foreign bonds, because of their low yield, are now less attractive.

In equilibrium, $R = R_C$, namely the return on government bonds in the good state equals the return on the corporate loan in the good state, regardless on the level of sovereign risk. As the limited liability constraint binds in the bad state, banks only care about the good state and their investment choice is not directly affected by the recovery rates in the bad state. Hence, in equilibrium, banks are not compensated to hold a higher risk in their loan to the productive sector nor in their holdings of government bonds.

3.3.3 Comparing Candidate Equilibria

I now compare quantities and prices in the candidate equilibria where both financial sectors are well capitalized (WW case) with the candidate equilibria where at least one financial sector is undercapitalized (UU, UW, WU cases).

In the WW case, following Section 3.3.1, the two financial sectors are identical and candidate equilibrium quantities and prices are $w = \{\alpha_W, k_W, R_W, R_C^W, D_W\}$. In the UU, UW, WU cases, following Section 3.3.2, the two bond markets clear with domestic
demand equal to domestic supply; as the public debt issuance in each country is entirely held by domestic banks, candidate equilibrium quantities and prices only depend on the capitalization of the domestic banks. In particular, in the UU, UW, WU cases, candidate equilibrium quantities and prices are $w' = \{1, k_W, R_W, R^C_W, D_W\}$ in a country with well capitalized domestic banks and $u = \{1, k_U, R_U, R^C_U, D_U\}$ in a country with undercapitalized domestic banks. While the home bias in the government bond portfolio only depends on banks’ capitalization (indeterminate in the WW case, equal to one in the UU, UW, WU cases), other equilibrium quantities and prices also depend on the level of sovereign risk.

Lemma 3. If $\gamma > \bar{\gamma}$, $k_W > k_U$, $R_W > R_U$, $R_W^C < R_U^C$, and $D_W < D_U$.

The comparison of quantities and prices in the equilibrium where domestic banks are well capitalized with quantities and prices in the equilibrium where domestic banks are undercapitalized depends on the level of sovereign risk. Suppose the sovereign risk is high ($\gamma > \bar{\gamma}$). In a country with well capitalized banks, domestic government bonds promise a higher return than the corporate loan as banks need to be compensated to hold the high sovereign risk. That’s not the case if the country’s banks are undercapitalized ($R_U = R_U^C$ in equilibrium) as banks do not need to be compensated for the defaults in the bad state—they only care about the good state. Hence, a high-sovereign risk country with undercapitalized banks has lower sovereign bond rates and higher corporate loan rates compared with the case where its banks are well capitalized.

The higher corporate loan rate causes the productive sector to borrow less from banks, reducing its investment in the productive technology. The resulting lower tax collection reduces the sovereign debt capacity, but is offset by the positive effect of the lower sovereign bond rate. This lower sovereign rate has a positive effect on the sovereign debt capacity as banks expect a higher sovereign recovery value in the bad state. Hence, in a country with high sovereign risk, (i) entrepreneurs invest less in their productive technology ($k_U < k_W$) and pay higher corporate loan rates ($R_U^C > R_W^C$) and (ii) the sovereign has a higher debt
capacity \((D_U > D_W)\) and pays lower sovereign rates \((R_U < R_W)\) if domestic banks are undercapitalized compared with the case where domestic banks are well capitalized.

### 3.4 Endogenous Limited Liability Constraint

Until now, I labeled banks as undercapitalized or well capitalized based on whether the limited liability constraint binds in the bad state. But, of course, the level of bank debt that triggers the limited liability constraint to bind depends on equilibrium quantities and prices.

The payoff in the bad state of a financial sector that invests its entire government bond portfolio domestically can be written as:

\[
\left[ \lambda_L (1 - k) R_d \right] \text{ post-haircut payoff of dom. govt bonds} + \lambda^C_L k R^C C \text{ post-haircut payoff of corp. loan} - L \right]^+\]

suggesting that there is a bank debt level threshold that determines whether a bank is well capitalized or undercapitalized. The following proposition formalizes the intuition:

**Proposition 3.** There exists a threshold \(L^i\) such that banks in \(i \in I\) are undercapitalized if \(L^i > \bar{L}^i\) and well capitalized if \(L^i \leq \bar{L}^i\).

If the initial level of bank debt is greater than this threshold, the limited liability constraint binds in the bad state (banks are undercapitalized). If the initial level of debt is lower than this threshold, the limited liability constraint does not bind in the bad state (banks are well capitalized). An increase in sovereign risk \(\gamma\) has two opposite effects on the threshold \(\bar{L}\). On the one hand, a higher sovereign risk reduces the recovery rate of domestic government bonds in the bad state, making the financial sector more likely to hit the limited liability constraint. On the other hand, a higher sovereign risk is associated with a higher payoff from the corporate loan, making the financial sector less likely to hit the limited liability constraint.
3.5 Equilibrium Bank Debt

I now solve the problem of the government at $t = 0$. If the sovereign risk is high, governments face a trade-off. On the one hand, well capitalized banks lend more to the productive sector at lower rates, increasing the payoff generated by the productive technology. On the other hand, undercapitalized banks, driven by their risk-shifting motive, choose a high investment in domestic government bonds, lowering sovereign yields and therefore expanding the sovereign debt capacity that, in turn, supports the public good provision.

The first two terms of the government objective function (3) illustrate this trade-off. More formally, define the difference in the government payoff—absent the cost of bank failure—between the candidate equilibrium with well capitalized domestic banks and the candidate equilibrium with undercapitalized domestic banks as follows:

$$\Delta W := \left( \Pi^P(k_W) - \Pi^P(k_U) \right) - \left( g(D_U) - g(D_W) \right)$$

where, if the sovereign risk is sufficiently high, following Lemma 3, the two terms in parentheses are strictly positive. The first term captures the higher payoff generated by the productive sector funded by a well capitalized domestic financial sector compared with the one funded by an undercapitalized financial sector. The second term captures the higher public good provision sustained by the demand for domestic bonds by undercapitalized domestic banks compared with well capitalized banks.

**Proposition 4.** Suppose $\gamma > \gamma$.

- If $\Delta W \geq 0$, $\alpha = \alpha_W$, $k = k_W$, $R = R_W$, $R^C = R^C_W$, $D = D_W$, $L \in [0, L]$.
- If $\Delta W < 0$, $\alpha = 1$, $k = k_U$, $R = R_U$, $R^C = R^C_U$, $D = D_U$, $L = \overline{L}$.

The equilibrium bank debt $L$ that maximizes the government payoff depends on $\Delta W$. 

20
If $\Delta W \geq 0$, the lower payoff generated by the productive sector funded by undercapitalized domestic banks offsets the benefit of higher public spending. The government therefore chooses $L \in [0, \bar{L}]$ and domestic banks never default. If $\Delta W < 0$, the benefit of higher public spending offsets the lower payoff generated by the productive sector funded by undercapitalized banks. The government therefore chooses $L = \bar{L}$ to minimize the cost of bank default (increasing in $L$) while still inducing a high demand for domestic government bonds by domestic banks.

The government objective function can also accommodate the fact that, in an environment with high sovereign risk, the marginal benefit of spending is likely high (Auerbach and Gorodnichenko, 2012). For example, the benefit of public spending, captured by the function $g$, can depend on the level of sovereign risk $\gamma$.\(^8\) This realistic feature would create an additional reason for a government with high sovereign risk to prefer undercapitalized domestic banks, as these would sustain the public good provision when it is needed the most. In Section 4.3, I discuss the case where the government also chooses the tax rate $\tau$.

### 4 Discussion

This section offers a discussion of the model assumptions, applications, and extensions. In Section 4.1, I discuss the role of the key assumptions. In Section 4.2, I discuss how the model insights might apply to the eurozone. In Section 4.3, I discuss a few model extensions.

---

\(^8\) $g$ would be a function of public debt $D$ and sovereign risk $\gamma$, where $\frac{\partial g}{\partial \gamma} > 0$, $\frac{\partial g}{\partial D} > 0$, and $\frac{\partial g}{\partial D \partial \gamma} > 0$. 
4.1 Assumptions

The first key assumption in the model is a sufficiently high correlation between the payoff of the productive technology and the payoff of domestic government bonds. Given that the sovereigns collect taxes only on the payoff of the domestic productive technology, the sovereigns mechanically default only if the domestic technology ends in the bad state. In this case, the low payoff reduces the tax collection triggering, in turn, the domestic sovereign default. As the shocks hitting the domestic and foreign productive technologies are uncorrelated, domestic and foreign sovereign defaults are also uncorrelated. In Section 4.3, I explain that the proposed risk-shifting mechanism survives (i) if the two shocks hitting the two productive technologies are not perfectly correlated, (ii) if sovereigns collect taxes also on the return to investing in sovereign bonds (as long as banks allocate a sufficiently large share of their sovereign bond holdings domestically), and (iii) if banks can also lend to the foreign productive sector (as long as banks allocate a sufficiently large share of their corporate credit domestically).\footnote{Backed by data, the home bias in banks’ holdings of government bonds and in banks’ private lending can be motivated by information frictions and the need to monitor borrowers.}

The second key assumption is a fixed bank balance sheet size that links the holdings of government bonds to the lending to the productive sector. Note that, in an environment with frictionless markets, large holdings of sovereign debt do not necessarily imply low private credit, as banks can finance government bonds with more borrowing. As in Gennaioli et al. (2014), financial frictions that constrain private borrowing motivate this assumption.

The third key assumption is the presence of a credible deposit insurance that protects depositors in the bad state. This guarantee must be realistically funded by a supranational body, as a nationally funded guarantee is unlikely to have sufficient funds to protect bank
depositors in case of a sovereign default. Thanks to this guarantee, depositors do not require a high return on their savings, nor do they discipline banks by withdrawing their deposits in bad times.\textsuperscript{10} Contrary to the literature on the bank-sovereign nexus, I decide to not model guarantees and instead focus on how bank debt affects bank portfolio choice and the payoff of the government. A supranational deposit insurance is not the only assumption that can sustain the proposed risk-shifting mechanism. Alternatively, I can assume that depositors can withdraw their savings and invest \textit{at a cost} in foreign assets. In this environment, banks and governments can take advantage of depositors—who have a “hold-up” problem—up to the cost of investing abroad.

\subsection*{4.2 Insights for the Eurozone}

The insights of the model can be applied to the eurozone because of two distinctive features of the eurozone setting: (i) the high covariance between bank and sovereign risks and (ii) the role of the ECB as a supranational safety net for banks.

First, eurozone banks’ risk of default tends to be particularly highly correlated with the domestic sovereign risk. Brunnermeier et al. (2016) argue that this correlation is mainly caused by the absence of a eurozone-wide safe asset and propose the issuance of “European Safe Bonds”—senior tranches of a well-diversified portfolio of eurozone sovereign bonds. The creation of this eurozone-wide safe asset would weaken the sensitivity of banks’ sovereign debt portfolios to domestic sovereign risk, especially in “peripheral” countries (Greece, Ireland, Italy, Portugal, Spain). In these countries, several banks have “solvency-critical” exposures toward the (risky) domestic sovereign. In Table 1, I show the sovereign exposures $\text{Exposure}_{ij}$ of bank $i$ vis-à-vis sovereign $j$ in December 2010. The first line shows the exposure of

\textsuperscript{10}See, for example, Black et al. (1978) for a discussion of deposit insurance and bank risk taking.
<table>
<thead>
<tr>
<th>Bank $i$ Country</th>
<th>Bank $i$ Name</th>
<th>Country $j$</th>
<th>$\text{Exposure}_{ij}$</th>
<th>$\text{Exposure}_{ij}/E_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR</td>
<td>ATEbank</td>
<td>GR</td>
<td>7,850</td>
<td>8.46</td>
</tr>
<tr>
<td>GR</td>
<td>TT Hellenic Postbank</td>
<td>GR</td>
<td>5,313</td>
<td>4.34</td>
</tr>
<tr>
<td>GR</td>
<td>Piraeus Bank</td>
<td>GR</td>
<td>8,114</td>
<td>2.30</td>
</tr>
<tr>
<td>IT</td>
<td>Monte dei Paschi</td>
<td>IT</td>
<td>32,018</td>
<td>2.26</td>
</tr>
<tr>
<td>ES</td>
<td>Caja Espana</td>
<td>ES</td>
<td>7,557</td>
<td>2.05</td>
</tr>
<tr>
<td>ES</td>
<td>Caixa</td>
<td>ES</td>
<td>34,332</td>
<td>1.82</td>
</tr>
<tr>
<td>GR</td>
<td>EFG Eurobank Ergasias</td>
<td>GR</td>
<td>8,740</td>
<td>1.55</td>
</tr>
<tr>
<td>ES</td>
<td>BBVA</td>
<td>ES</td>
<td>53,451</td>
<td>1.43</td>
</tr>
<tr>
<td>GR</td>
<td>National Bank of Greece</td>
<td>GR</td>
<td>12,883</td>
<td>1.38</td>
</tr>
<tr>
<td>ES</td>
<td>CAM</td>
<td>ES</td>
<td>5,587</td>
<td>1.36</td>
</tr>
<tr>
<td>PT</td>
<td>Banco BPI</td>
<td>PT</td>
<td>3,896</td>
<td>1.34</td>
</tr>
<tr>
<td>IT</td>
<td>Intesa Sanpaolo</td>
<td>IT</td>
<td>57,622</td>
<td>1.32</td>
</tr>
<tr>
<td>ES</td>
<td>Unnim Caixa</td>
<td>ES</td>
<td>2,574</td>
<td>1.22</td>
</tr>
<tr>
<td>ES</td>
<td>Banco de Sabadell</td>
<td>ES</td>
<td>7,296</td>
<td>1.16</td>
</tr>
<tr>
<td>IT</td>
<td>Banco Popolare</td>
<td>IT</td>
<td>11,759</td>
<td>1.16</td>
</tr>
<tr>
<td>ES</td>
<td>Banco Pastor</td>
<td>ES</td>
<td>2,183</td>
<td>1.11</td>
</tr>
<tr>
<td>IE</td>
<td>Irish Life and Permanent</td>
<td>IE</td>
<td>1,852</td>
<td>1.10</td>
</tr>
<tr>
<td>ES</td>
<td>BFA-Bankia</td>
<td>ES</td>
<td>25,382</td>
<td>1.05</td>
</tr>
</tbody>
</table>

**Table 1: Peripheral Banks’ Sovereign Exposures.** The first and second column report the banks' country of incorporation and bank name. The third column shows the sovereign with respect to which the exposure ($\text{Exposure}$) is measured. The last two columns report $\text{Exposure}$ (million euro) and $\text{Exposure}/E$, where $E$ is total capital (total own funds for solvency purpose, sum of tier 1, tier 2, and tier 3 capital). $\text{Exposure}$ is the sovereign exposure net of cash short positions as of December 31, 2010. Banks are ranked according to $\text{Exposure}/E$. Only banks with $\text{Exposure}/E > 1$ are reported. Source: EBA, Bureau van Dijk.

ATEbank (Greek bank) vis-à-vis Greece. I then rank banks according to their $\text{Exposure}_{ij}/E_i$ ratio, where $E_i$ is bank $i$ total capital. Of the 1,420 \{i, j\} pairs, the table only shows the pairs where the ratio is greater than one, suggesting that bank $i$ exposure to country $j$ is solvency-critical. Note that all the reported exposures are domestic, and 18 out of the 43 peripheral banks in the sample have solvency-critical exposures.

Second, starting in 2008, the ECB has effectively provided a credible safety net for eurozone banks, reducing the threat of runs by depositors. Starting in October 2008, banks could obtain unlimited liquidity from the ECB provided they pledged eligible collateral. Crucially, eligible collateral included low-quality and illiquid assets, allowing banks to effectively shift
the risk of some of the potential sovereign default losses to the central bank (Uhlig, 2013).\footnote{García-de Andoain et al. (2016) shows that peripheral banks relied on the ECB as a funding source. In addition, euro-denominated government bonds have a zero regulatory risk weight for eurozone banks. Note that the zero regulatory risk weight regime does not explain the preference for domestic over foreign euro-denominated government bonds. See the Capital Requirement Directive (Directive 2006/48/EC, Annex VI, Part 1(4)).}

### 4.3 Extensions

The model can be extended in several dimensions. First, the risk-shifting mechanism survives if the shocks hitting the two productive technologies are correlated unless they are perfectly correlated. The non-zero correlation affects the portfolio choice of undercapitalized banks that make their investment decisions to maximize the payoff in the good state. In this state, the domestic sovereign never defaults, but the foreign sovereign might default if the shocks hitting the productive technologies are not perfectly correlated. If they are perfectly correlated, the two governments always default at the same time.

Second, the risk-shifting mechanism survives if I allow sovereigns to collect taxes also on the return to investing in sovereign bonds, as long as banks allocate a sufficiently large share of their sovereign bond holdings domestically. If banks’ investment in foreign bonds is large enough, the domestic tax collection will partly depend on the foreign state of the world, weakening the correlation between the payoff from private lending and the payoff from domestic government bonds. If banks allocate a sufficiently large share of their sovereign bond holdings domestically, undercapitalized banks—that have an incentive to tilt their government bond portfolio toward the asset that pays the most in the good state—invest in domestic bonds as their return in the good state is higher than that of foreign bonds.

Third, the risk-shifting mechanism survives if I allow banks to also lend to the foreign productive sector, as long as banks allocate a sufficiently large share their corporate credit
domestically. However, banks with a sufficiently high lending to the foreign productive sector default when the foreign economy is in the bad state, weakening the positive correlation between the payoff from private lending and the payoff from domestic government bonds. Similar to the previous case, undercapitalized banks invest in domestic government bonds if their return in the good state is higher than the return of foreign government bonds.

Fourth, the model can be extended to allow governments to choose the tax rate \( \tau \). In this case, governments can use the tax rate to affect the investment choice of the productive sector and, therefore, tilt the portfolio allocation of banks between investing in sovereign bonds and lending to the productive sector. Governments face a trade-off in their tax policy. On the one hand, governments would like to tax the productive sector to have some debt capacity to support the provision of the public good. On the other hand, governments would like to set a low tax rate to induce a high investment in the productive technology so that banks lend most of their balance sheet to the productive sector in equilibrium. In this case, banks would invest little in the sovereign bond market and sovereigns would have limited debt capacity, limiting the effect of the (sovereign) shock to their tax collection.

5 Empirical Evidence

In this section, I present empirical evidence consistent with bank leverage playing an important role in determining bank holdings of risky domestic government bonds—the main model prediction. The evidence is based on the 2010 eurozone stress test, undertaken by the European Banking Authority (EBA) before the worsening of the sovereign crisis in the second half of 2011. The data is publicly available on the website of the EBA (www.eba.europa.eu).

In December 2010, peripheral banks held more domestic government bonds, driven by low-capital banks, compared with non-peripheral banks. In December 2010, banks in peripheral countries held 8.1% of total assets in domestic government bonds, equivalent to 90% of their entire government bond portfolio. Banks in low-risk countries held only 1.9% of
total assets in peripheral government bonds, equivalent to 12.8% of their government bond portfolio, and 10% of total assets in (safe) domestic government bonds, equivalent to 52% of their government bond portfolio. In peripheral countries, the high holdings of domestic sovereign bonds were driven by low-capital banks. The correlation between holdings of domestic government bonds (share of total assets) and leverage (ratio of capital to total assets) in the sample of the 43 stress-tested peripheral banks was -0.27, with a 0.08 p-value.

6 Conclusion

In this paper, I build a tractable model where the sovereign debt capacity depends on the capitalization of domestic banks. I show that undercapitalized banks hold domestic government bonds to link their destiny to that of the sovereign. The rationale is risk-shifting. While in the case of domestic sovereign default banks are protected by limited liability, the home sovereign debt guarantees a high payoff in the good state of the world. If the sovereign risk is sufficiently high, undercapitalized banks lend less to the productive sector to hold even more domestic government bonds. This increased demand for domestic government bonds reduces sovereign yields therefore supporting the sovereign debt capacity.

The model suggests that national regulators face a trade-off when setting capital requirements in an environment with high sovereign risk. On the one hand, well capitalized banks lend to firms and households fostering growth. On the other hand, low-capital banks optimally act as buyers-of-last-resort for the home sovereign sustaining its debt capacity. If the sovereign risk is sufficiently high, sovereigns with undercapitalized domestic banks have a higher debt capacity and pay lower sovereign yields compared with sovereigns with well capitalized domestic banks.

The analysis in this paper warns against establishing a supranational deposit insurance in the presence of undercapitalized and geographically undiversified banks. In this environment, in countries with high sovereign risk, low-capital banks might hold more domestic government
bonds to risk-shift on their insured deposit holders reducing, in turn, their credit supply to firms. This behavior might help governments access public debt markets at a lower cost, but it also exacerbates the sovereign-bank nexus. When in place, an international safety net should be paired with careful bank supervision to preserve financial stability. I believe these are promising avenues for future research.

References


Appendix

Proof of Lemma 1. The first order condition of the entrepreneurs’ problem is:

\[ k = \left( \frac{\mathbb{E}(\epsilon)(1 - \tau)}{2\mathbb{E}(\lambda^C)R^C} \right)^2 \]  

(A1)

At \( t = 1 \), the entrepreneurs’ payoff is \((1 - \tau)\epsilon_s\sqrt{k} \) for \( s = \{H, L\} \) and entrepreneurs need to repay \( kR^C \). Rearranging, the entrepreneurs default iff \( 2\epsilon_s\mathbb{E}(\lambda^C) < \mathbb{E}(\epsilon) \) for \( s = \{H, L\} \).

If \( \theta\epsilon_H > \epsilon_L(1 + \theta) \) holds, the entrepreneurs default in the bad state as \( 2\epsilon_L\mathbb{E}(\lambda^C) < \mathbb{E}(\epsilon) \).

The recovery rate can be expressed as \( \lambda^C_L = \frac{2\epsilon_L\mathbb{E}(\lambda^C)}{\mathbb{E}(\epsilon)} \) or \( \lambda^C_L = \frac{2\theta\epsilon_L}{\theta\epsilon_H - (1 - \theta)\epsilon_L} \lambda^C_H \). Suppose the entrepreneurs also default in the good state. The recovery rate \( \lambda^C_H \) is such that \( 2\epsilon_H\mathbb{E}(\lambda^C) = \lambda^C_H\mathbb{E}(\epsilon) \). Plugging in \( \mathbb{E}(\lambda^C) \) and \( \lambda^C_H \) as a function of \( \lambda^C_L \), I reach a contradiction. Hence, the entrepreneurs do not default in the good state (\( \lambda^C_H = 1 \)) and

\[ \lambda^C_L = \frac{2\theta\epsilon_L}{\theta\epsilon_H - \theta\epsilon_L - \epsilon_L} \in (0, 1) \]  

(A2)

It is easy to verify that there is no default in the good state, i.e. \( 2\epsilon_H\mathbb{E}(\lambda^C) > \mathbb{E}(\epsilon) \). \( \square \)

Proof of Lemma 2. The payment due to bondholders at \( t = 1 \) is \( DR = \mathbb{E}(\epsilon)\tau\sqrt{k} - y \). In the good state, the government is able to fully repay bondholders as \( \tau\epsilon_H\sqrt{k} - y > \tau\mathbb{E}(\epsilon)\sqrt{k} - y \). In the bad state, the government defaults on part of its debt as \( \tau\epsilon_L\sqrt{k} - y < \tau\mathbb{E}(\epsilon)\sqrt{k} - y \).

\( \lambda_L \) is such that tax collection equals the payment due to bondholders:

\[ \lambda_L = \frac{\tau\epsilon_L\sqrt{k} - y}{\tau\mathbb{E}(\epsilon)\sqrt{k} - y} \in (0, 1) \]  

(A3)

\( \square \)

Proof of Proposition 1. Given (4) and (A1), the two financial sectors must invest in both corporate credit and the sovereign bond market for the two sovereign markets and the two corporate loan markets to clear. In equilibrium, \( R\mathbb{E}(\lambda) = R^s\mathbb{E}(\lambda^s) = R^C\mathbb{E}(\lambda^C) \). Market
clearing also implies that $\alpha^A = \alpha^B$.

**Closed-Form Expressions** Using (A1) and bond market clearing, I obtain:

$$
\mathbb{E}(\lambda^C) R_W^C = \mathbb{E}(\lambda) R_W = \frac{1}{2} \sqrt{\mathbb{E}(\epsilon)(1 - \tau)(\mathbb{E}(\epsilon)(1 - \tau) + 2\tau \Delta \mathbb{E}(\lambda))},
$$

(A4)

$$
k_W = \frac{\mathbb{E}(\epsilon)(1 - \tau)}{\mathbb{E}(\epsilon)(1 - \tau) + 2\tau \Delta \mathbb{E}(\lambda)} \in (0, 1), \quad (A5)
$$

where $\lambda_L = \epsilon_L(1 - \gamma)\Delta^{-1} \in (0, 1)$, $\Delta = \theta \epsilon_H + (1 - \theta - \gamma)\epsilon_L$, and $\gamma$ is such that $y = \gamma \tau \epsilon_L \sqrt{k}$.

Plugging (A5) in $y = \gamma \tau \epsilon_L \sqrt{k}$, it is easy to show that $\frac{\partial y}{\partial \gamma} > 0$. Given that $y(\gamma)$ is one-to-one, the inverse is also strictly positive. Recall that we focus on the case where $y < \tau \epsilon_L \sqrt{k}$.

**Proof of Proposition 2.** From banks’ problem (2) and Lemma 2, if at least one financial sector is undercapitalized, market clearing in the bond market implies that $\alpha^A = \alpha^B = 1$. Banks require $R = R^C$ to invest in both government bonds and corporate credit.

**Closed-Form Expressions** Using (A1) and bond market clearing:

$$
R_U = R^C_U = \frac{1}{2\mathbb{E}(\lambda^C)} \sqrt{\mathbb{E}(\epsilon)(1 - \tau)(\mathbb{E}(\epsilon)(1 - \tau) + 2\tau \Delta \mathbb{E}(\lambda^C))}, \quad (A6)
$$

$$
k_U = \frac{\mathbb{E}(\epsilon)(1 - \tau)}{\mathbb{E}(\epsilon)(1 - \tau) + 2\tau \Delta \mathbb{E}(\lambda^C)} \in (0, 1), \quad (A7)
$$

where $\lambda_L = \epsilon_L(1 - \gamma)\Delta^{-1} \in (0, 1)$, $\Delta = \theta \epsilon_H + (1 - \theta - \gamma)\epsilon_L$, and $\gamma$ is such that $y = \gamma \tau \epsilon_L \sqrt{k}$.

Plugging (A7) in $y = \gamma \tau \epsilon_L \sqrt{k}$, it is easy to show that $\frac{\partial y}{\partial \gamma} > 0$. Given that $y(\gamma)$ is one-to-one, the inverse is also strictly positive. Recall that we focus on the case where $y < \tau \epsilon_L \sqrt{k}$.

**Remark** Using (A2) and (A3), $\lambda_L < \lambda^C_L$ iff $\gamma > \overline{\gamma}$, where $\overline{\gamma} = 1 - \frac{2\theta^2 (\epsilon_H - \epsilon_L)}{\theta \epsilon_H - \epsilon_L(1 + \theta)}$. Hence, $R_W > R^C_W$ iff $\gamma > \overline{\gamma}$.

**Proof of Lemma 3.** Using (A2) and (A3), we find that $\lambda^C_L > \lambda_L$ iff $\gamma > \overline{\gamma}$, where $\overline{\gamma} = \frac{2\theta^2 (\epsilon_H - \epsilon_L)}{\theta \epsilon_H - \epsilon_L(1 + \theta)}$. Hence, $R_W > R^C_W$ iff $\gamma > \overline{\gamma}$. 

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1 − \frac{2\theta^2(\epsilon_H - \epsilon_L)}{\theta \epsilon_H - \epsilon_L(1 + \theta)}. If \gamma > \bar{\gamma}, using (A5) and (A7), k_W > k_U and, therefore by market clearing, \(D_W < D_U\). Hence, using the sovereign debt capacity, \(R_U < R_W\). It follows that \(R_C^U > R_C^W\).

**Proof of Proposition 3.** Banks’ payoffs in the good and bad state with \(\alpha^A = \alpha^B\) are:

\[
\Pi_H(k)\big|_{\alpha^A = \alpha^B} = kR_C + R(1 - k) \quad \text{and} \quad \Pi_L(k)\big|_{\alpha^A = \alpha^B} = k\lambda_L R_C + R\lambda_L(1 - k)
\]

where \(\Pi_L(k)\big|_{\alpha^A = \alpha^B} < \Pi_H(k)\big|_{\alpha^A = \alpha^B}, \forall k > 0\). The unconstrained problem of a well capitalized financial sector has solution \(k^* \in (0,1)\) and the constrained problem of an undercapitalized financial sector has solution \(k^{**}\). Define \(\bar{k}\) be such that \(\Pi_L(k)\big|_{\alpha^A = \alpha^B} = L\).

There are four cases: (i) if \(k^* \geq \bar{k}\) and \(k^{**} \geq \bar{k}\), the solution is \(k^*\) as:

\[
\mathbb{E}(\Pi(k^*)) - L = \theta \Pi_H(k^*) - L\theta + (1 - \theta)\Pi_L(k^*) - L(1 - \theta) \\
\geq \theta \Pi_H(k^{**}) - L\theta + (1 - \theta)\Pi_L(k^*) - L(1 - \theta) \geq \theta \Pi_H(k^{**}) - L\theta
\]

(ii) if \(k^* < \bar{k}\) and \(k^{**} < \bar{k}\), the solution is \(k^{**}\) as:

\[
\theta \Pi_H(k^{**}) - L\theta \geq \theta \Pi_H(k^*) - L\theta \\
\geq \theta \Pi_H(k^*) - L\theta + (1 - \theta)\Pi_L(k^*) - L(1 - \theta) = \mathbb{E}(\Pi(k^*)) - L
\]

(iii) if \(k^* \geq \bar{k}\) and \(k^{**} < \bar{k}\), the solution is \(k^*\) as:

\[
\mathbb{E}(\Pi(k^*)) - L = \theta \Pi_H(k^*) - L\theta + (1 - \theta)\Pi_L(k^*) - L(1 - \theta) \\
\geq \theta \Pi_H(k^{**}) - L\theta + (1 - \theta)\Pi_L(k^*) - L(1 - \theta) \geq \theta \Pi_H(k^{**}) - L\theta
\]

(iv) if \(k^* \leq \bar{k}\) and \(k^{**} > \bar{k}\), the solution is \(k^{**}\) as:

\[
\theta \Pi_H(k^{**}) - L\theta + (1 - \theta)\Pi_H(k^{**}) - L(1 - \theta) \geq \theta \Pi_H(k^{**}) - L\theta \geq \theta \Pi_H(k^*) - L\theta
\]

Hence, the solution is \(k^*\) if \(k^* \geq \bar{k}\) and \(k^{**}\) if \(k^* < \bar{k}\) ad \(\bar{L}\) is such that \(\bar{L} = (1 - \tau\gamma)\epsilon_L \sqrt{k_W}\). □

**Proof of Proposition 4.** The proposition follows from (3) and Lemma 3. □